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Effect of porosity and stress concentration associated with pores on elastic creep by crack growth in brittle solids

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The solution for the creep rate of a porous solid as a function of pore volume fraction, pore size and the radial or annular flaw size, is presented. The analysis is based on the concept of crack opening displacement which assumes that the displacement of a solid under stress is solely due to the opening of all radial or annular cracks emanating from the surface of the cavities. It is shown that the strain rate by crack growth is a strong function of pore size and preexisting flaw size. A linear relationship was found to exist between the strain rate by crack growth and the porosity volume fraction.

1. Introduction

Brittle solids are known to fracture catastrophically when applied stress reaches a critical value. Stresses below this critical value may also cause fracture if applied and maintained constant for a sufficient length of time. This time-dependent deformation is referred to as creep. The creep deformation of solids can occur by a number of mechanisms such as dislocation climb [1, 2], dislocation glide [3, 4], grain boundary sliding [5, 6] and Coble [9] creep.

Recently, it has been proposed that an additional mechanism of creep deformation of ceramic material can be operative over ranges of stress and temperature at which other creep mechanisms are not expected to be dominant and is referred to as "elastic" or compliance creep [10, 11]. This mechanism is based on microstructural changes occurring during creep deformation and can be superimposed on any other creep process. Extensive experimental studies of the creep kinetics have indicated that crack nucleation and growth are solely responsible for elastic creep in a wide variety of ceramics [12–14].

An important observation made in a number of ceramic materials is that this non-linear creep is most frequently observed in solids with a large grain size or with the presence of residual pore phase [15, 16]. Based on the results obtained from the study of the effect of microstructure on creep deformation in polycrystalline alumina, Crosby and Evans [16] suggested that pores play an active role in promoting elastic creep by crack growth.

The purpose of this paper is to present an analysis on the effect of pores and the stress concentration induced by the presence of pores on the elastic creep of ceramics occurring at low and intermediate temperatures. A concept of crack opening displacement is introduced to model the strain rate of a solid containing pores.

2. Elastic creep based on crack opening displacement

It is well known that most ceramics contain pores even in the fully dense state produced by high-pressure high-temperature techniques. The presence of pores invariably leads to the generation of stress concentrations which are often so high that they can not be neglected. It has been demonstrated in a number of systems that the presence of pores plays a dominant role in determining the mechanical properties and elastic response of solids when subjected to external stress [17–19]. It is further believed that pores also influence the creep deformation of a structural component by the growth of radial cracks and their excessive opening due to the presence of stress concentrations. In what follows it is assumed that the total strain of a solid containing cavities is entirely due to opening of these cavities and cracks associated with cavities. The contribution of all other deformational mechanisms is assumed to be absent. As a consequence, this mechanism of time-dependent deformation is expected to be operative in a high-stress low-temperature regime where blunting effects due to mass transport are minimal.

Further, it is assumed that each cavity possesses an annular or radial crack on both sides as shown in Fig. 1. When an external stress is applied on such a solid, it develops a stress concentration outside the cavity and this stress can be represented by a tangential component of the form [20] (for

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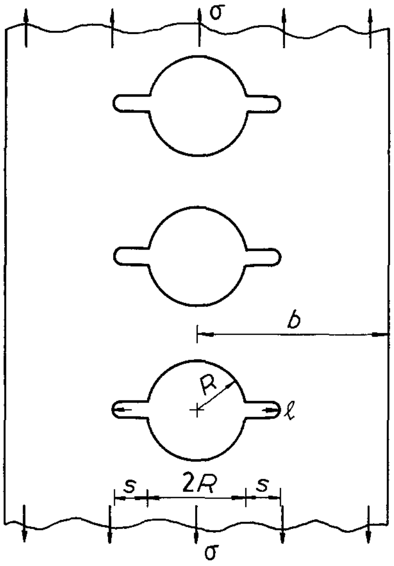

Figure 1 Spherical pores subjected to external stress.

$\theta = \pi/2$).

$$\sigma_\theta = \sigma\left[\frac{4-5\nu}{2(7-5\nu)}\left(\frac{R}{x}\right)^3 + \frac{9}{2(7-5\nu)}\left(\frac{R}{x}\right)^5 + 1\right] \tag{1}$$

for a spherical pore, and

$$\sigma_\theta = \sigma\left[\frac{1}{2}\left(\frac{R}{x}\right)^2 + \frac{3}{2}\left(\frac{R}{x}\right)^4 + 1\right] \tag{2}$$

for a cylindrical cavity. In Equations 1 and 2 σ is the applied stress, ν Poisson's ratio, R the cavity radius and x the distance from the centre of the cavity.

For a cylindrical cavity with radial cracks on both sides (Fig. 1), the total displacement (Δ_t) at remote points is [21]

$$\Delta_t = \Delta_{nc} + \Delta_c \tag{3}$$

where Δ_{nc} represents the displacement of a solid containing no cracks and Δ_c is the displacement due to crack opening.

The displacement due to crack opening is given by the expression [21]

$$\Delta_c = \frac{4\sigma C}{E'} V_2 \tag{4}$$

where

$$V_2 = -1.071 + 0.250\left(\frac{C}{b}\right) - 0.357\left(\frac{C}{b}\right)^2 + 0.121\left(\frac{C}{b}\right)^3 - 0.047\left(\frac{C}{b}\right)^4 + 0.008\left(\frac{C}{b}\right)^5 - 1.071\frac{1}{C/b}\ln\left(1-\frac{C}{b}\right)$$

C ($= R + s$) is the total crack length, σ the applied stress, b the sample width and

$$E' = E_0 \quad \text{for plane stress}$$

$$E' = E/(1-\nu^2) \quad \text{for plane strain}$$

For the case of a single isolated crack in a large specimen ($C/b \to 0$) Equation 4 assumes a simple form

$$\Delta_c = \frac{3.144\sigma C}{E'}\left(\frac{C}{b}\right) \tag{5}$$

The displacement due to the opening of N cracks is

$$\Delta_c = \frac{3.144\sigma N C^2}{bE'} \tag{6}$$

The compliance of a solid of height, h, and width, b, with N^c ($= N/bh$) cracks (cavities) per unit area is

$$Q_{ef} = \frac{\Delta_t}{\sigma h} = \left(\frac{1}{E_0} + \frac{3.144 N^c C^2}{E'}\right) \tag{7}$$

and the total strain is

$$\varepsilon_c = \sigma Q_{ef} = \left(\frac{\sigma}{E_0} + \frac{3.144\sigma N^c C^2}{E'}\right) \tag{8}$$

The maximum crack opening and hence the maximum displacement at remote points due to a single isolated circular crack can be obtained through the expression relating the compliance, Q_{ef}, and the strain energy release rate, G,

$$G = \frac{P^2}{2}\frac{\partial Q_{ef}}{\partial A} \tag{9}$$

where P is the applied load, A is the crack surface area and Q_{ef} is now defined as displacement over force.

The stress intensity factor (K_I) for a circular crack of length C located in the centre of a cylindrical specimen of radius b is [21]

$$K_I = \sigma_{net}(\pi C)^{1/2} V_3 \tag{10}$$

where

$$V_3 = \frac{2}{\pi}\left[1 + \frac{1}{2}\left(\frac{C}{b}\right) - \frac{5}{8}\left(\frac{C}{b}\right)^2 + 0.421\left(\frac{C}{b}\right)^3\right] \times \left[1 - \frac{C}{b}\right]^{1/2}$$

$$\sigma_{net} = \frac{P}{\pi(b^2 - C^2)}$$

$$\sigma = \frac{P}{\pi b^2}$$

$$A = \pi C^2$$

$$dA = 2\pi C dC$$

Neglecting the terms beyond (C/b) for $C/b \to 0$, the compliance of a solid containing a single isolated penny shaped crack can now be obtained by combining Equations 9 and 10 to yield

$$dQ_{ef} = \frac{16}{\pi^2 bE'}\left(\frac{C}{b}\right)^2 d\left(\frac{C}{b}\right) \tag{11}$$

Integration of Equation 11 gives

$$Q_{ef} = \frac{16}{3\pi^2 bE'}\left(\frac{C}{b}\right)^3 \tag{12}$$

The integration of Equation 11 has been carried out assuming that Q_{ef} is zero so that the integrated value in Equation 12 represents the compliance due to the cracks only. The displacement and compliance are related via the equation

$$\Delta_t = Q_{ef}P = \frac{16\sigma C^3}{3\pi E' b^2} \tag{13}$$

For N^p cracks (pores) per unit volume ($N^p = N/\pi b^2 h$) and $\Delta_{nc} = (\sigma/E)h$, the total strain of a solid is

$$\frac{\Delta_t}{h} = \varepsilon_p = \sigma Q_{ef} = \sigma\left(\frac{1}{E_0} + \frac{16N^p C^3}{3E'}\right) \qquad (14)$$

For plane strain condition, Equation 14 may be written in the form

$$\varepsilon_p = \varepsilon_0 + \frac{16(1-\nu^2)\sigma N^p C^3}{3E_0} \qquad (15)$$

which recovers the result obtained by Hasselman and Singh [22]. In Equation 15, ε_0 is the elastic strain of a crack-free solid.

So far the analysis has concentrated on a solid containing large number of cracks subjected to a uniform external stress in which most of these cracks are located away from the specimen surface. In the case of annular or radial cracks emanating from the cavity surface, the stress concentration due to the presence of cavities is much larger than the applied stress (two or three times) and this is considered to have a significant effect on the crack extension process. The effect of stress concentration on crack opening displacement and therefore elastic strain of a solid, can be obtained by imposing a tangential component of stress concentration outside a spherical pore on the surface of an annular or radial crack. For the crack-cavity geometry depicted in Fig. 1, a tangential component is tensile in nature and is acting in the direction to open the crack. Thus, from Equations 1, 2, 8 and 15, the maximum strain due to opening of cracks in a porous solid is (for plane strain condition)

$$\varepsilon_c = \frac{3.144(1-\nu^2)\sigma N^c (R+s)^2}{E_0} \times \left(\frac{R^2}{2(R+s)^2} + \frac{3R^4}{2(R+s)^4} + 1\right) \qquad (16)$$

$$\varepsilon_p = \frac{16(1-\nu^2)\sigma N^p (R+s)^3}{3E_0}\left(\frac{4-5\nu}{2(7-5\nu)}\frac{R^3}{(R+s)^3} + \frac{9}{2(7-5\nu)}\frac{R^5}{(R+s)^5} + 1\right) \qquad (17)$$

It can easily be shown from Equations 16 and 17 that the corresponding strain rates by crack growth are

$$\frac{d\varepsilon_c}{dt} = \dot\varepsilon_c = 6.288(1-\nu^2)N^c R(1+s/R)\frac{\sigma}{E_0} \times \left(1 - \frac{3}{2(1+s/R)^4}\right)\frac{ds}{dt} \qquad (18)$$

$$\frac{d\varepsilon_p}{dt} = \dot\varepsilon_p = 16(1-\nu^2)N^p R^2(1+s/R)\frac{\sigma}{E_0} \times \left(1 - \frac{3}{7-5\nu}\frac{1}{(1+s/R)^5}\right)\frac{ds}{dt} \qquad (19)$$

From Equations 18 and 19 it is immediately evident that, as expected, there is a strong s/R dependence of strain rate and this dependence can further be reinforced by the creation of new cracks which is a common feature occurring in many polycrystalline solids subjected to stress.

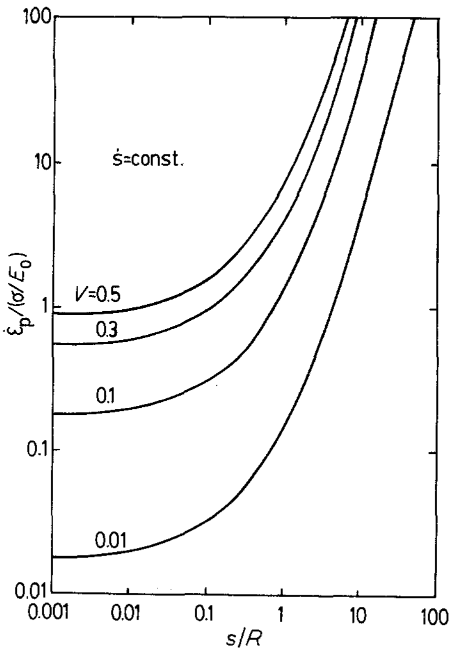

Figure 2 Predicted variation of a normalized strain rate with s/R at a given porosity volume fraction. $\dot s$ = constant.

Assuming the stress field interaction of neighbouring cracks is negligible (for small pore volume fractions), it is of interest to relate the pore volume fraction to the rate of elastic creep induced by the radial crack growth. The relationship between the number of pores per unit volume (N^p) and their volume fraction (V) is given by the expression

$$N^p = \frac{3V}{4\pi R^3} \qquad (20)$$

when N^p in Equation 19 is replaced by Equation 20, the creep rate of a solid containing pores becomes

$$\dot\varepsilon_p = \frac{12V\sigma}{RE_0}(1-\nu^2)(1+s/R)^2 \times \left(1 - \frac{3}{7-5\nu}\frac{1}{(1+s/R)^5}\right)\frac{ds}{dt} \qquad (21)$$

Fig. 2 illustrates the predicted change of strain rate with s/R for various porosity levels. Clearly, Equation 21 shows that the strain rate is inversly proportional to the pore size which means that, for a given volume fraction, a larger number of small size pores will exhibit lower strain rate in comparison to one containing a small number of large size pores. Another interesting result of Equation 21 is that at sufficiently high values of s/R (> 1) the effect of stress concentration due to the presence of pores becomes negligible. In such cases, the presence of pores affects the strain rate by crack growth only through the increase of total crack length ($R + s$) and the crack density.

For large s/R (> 1) for which the interaction effects between the pore stress field and the crack tip stress field are minimal, Equation 21 reduces to

$$\dot\varepsilon_p = \frac{12V\sigma}{\pi RE_0}(1-\nu^2)(1+s/R)^2\dot s \qquad (22)$$

where $\dot s = ds/dt$. Thus, the total strain rate of a solid containing pores is

$$\dot\varepsilon_t = \dot\varepsilon_0 + \frac{12V\sigma}{\pi RE_0}(1-\nu^2)(1+s/R)^2\dot s \qquad (23)$$

where $\dot\varepsilon_0$ is the elastic strain rate of a pore-free solid.

3. Effect of stress intensity factor on elastic creep

At low temperatures, where mass transport by diffusional mechanisms is absent, a radial crack extension will occur when a local stress intensity factor, K_I, at the tip of a crack reaches a critical value, K_IC. However, substantial crack growth can occur at stress intensity factors well below a critical value, and this type of crack extension was found to be a strong function of stress intensity factor. Extensive literature data have shown that the rate of crack growth at a given temperature can be described by the equation of the form

$$\dot{s} = AK_\mathrm{I}^n \tag{24}$$

where A and n are constants for a given material.

For an annular flaw emanating from the spherical pore, the stress intensity factor was found to be [19]

$$K_\mathrm{I} = 2\sigma R^{1/2}\left(\frac{1 + s/R}{\pi}\right)^{1/2}\phi \tag{25}$$

where

$$\begin{aligned}\phi = {} & 3\left[1 - \left(1 - \frac{1}{(1 + s/R)^2}\right)^{1/2}\right] \\ & + \left(1 - \frac{1}{(1 + s/R)^2}\right)^{1/2} \\ & \times \left(\frac{1}{2(1 + s/R)^2} + \frac{1}{2(1 + s/R)^4} + 1\right)\end{aligned}$$

Combination of Equations 24 and 25 gives

$$\dot{s} = A\,(2\sigma\phi)^n R^{n/2}\left(\frac{1 + s/R}{\pi}\right)^{n/2} \tag{26}$$

The creep rate by crack growth can now be obtained by substituting Equation 26 in Equation 19 to yield for a constant crack density, N^p

$$\begin{aligned}\dot{\varepsilon}_\mathrm{p} = {} & \frac{16A(1 - v^2)}{\pi^{n/2}E_0}(1 + s/R)^{n/2+1}(2\phi)^n N^\mathrm{p}\sigma^{n+1}R^{n/2+2} \\ & \times \left(1 - \frac{3}{(7 - 5v)(1 + s/R)^5}\right)\end{aligned} \tag{27}$$

Experimental data on a wide variety of polycrystalline ceramics have shown that n can vary from 10 for hot-pressed silicon nitride [23] to nearly 18 for soda-lime–silica glass [24], indicating that the elastic creep by crack growth can be highly non-linear. The same conclusion has been reached by Venkateswaran and Hasselman [10] who used a somewhat different approach.

The effect of pore volume fraction on creep rate can now be obtained by combining Equations 20 and 27 to give

$$\begin{aligned}\dot{\varepsilon}_\mathrm{p} = {} & \frac{12AV(1 - v^2)}{E_0\pi^{n/2+1}}(1 + s/R)^{n/2+1}(2\phi)^n\sigma^{n+1}R^{n/2-1} \\ & \times \left(1 - \frac{3}{(7 - 5v)(1 + s/R)^5}\right)\end{aligned} \tag{28}$$

Implicit in the use of Equation 28 is the premise that the stress concentration induced by the presence of pores affects both the rate of crack growth ($\dot{s}$) and the crack opening. If, however, it is assumed that the creep rate is dictated by the opening of N^p cracks per unit volume, without taking into account the effect of stress concentration due to the presence of pores, then from Equation 15 the elastic creep rate by crack growth is

$$\dot{\varepsilon}_\mathrm{p} = \frac{16(1 - v^2)N^\mathrm{p}\sigma C^2}{E_0}\dot{s} \tag{29}$$

Considering that the condition for crack extension is determined by the stress intensity factor at the tip of a crack, then the creep rate by crack growth becomes (combination of Equations 20, 26 and 29)

$$\dot{\varepsilon}_\mathrm{p} = \frac{12(1 - v^2)AV}{\pi^{n/2+1}E_0}(2\phi)^n\sigma^{n+1}R^{n/2-3}(1 + s/R)^{n/2+2} \tag{30}$$

Again, as in previous cases, the creep rate is highly non-linear.

4. Rate of crack growth by mass transport

The propagation of cracks by diffusional processes has been a matter of controversy for quite some time. Liu [25] has considered the strain field interaction between a crack tip stress field and a point defect. From this treatment it is concluded that, based on the first-order interaction effect, the vacancies are always repelled from the crack tip whereas interstitials are attracted to it. Heald *et al.* [26] on the other hand have shown that when the second-order interaction effect is considered, the crack tip always attracts vacancies but repels interstitials. This contradicts the stress corrosion cracking model developed by Liu. Crussard [27] was the first who pointed out that the second-order elastic interaction must be taken into account when considering the diffusion of point defects to a crack tip. According to Crussard vacancies will always diffuse towards the crack tip in agreement with Heald *et al.*

A somewhat more generalized approach to this problem has been developed by Stevens and Dutton [28] who calculated the total chemical potential of atoms at the crack tip by including the strain and surface energy terms. A corollary of their model is that vacancies will be either attracted to or repelled from a crack tip according to magnitude of the applied stress. From Stevens' and Dutton's analysis the crack growth by surface and grain boundary diffusion is

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \Omega^2 N_\mathrm{a} D_\mathrm{s}(B\sigma^2 - \gamma_\mathrm{gb}/C)/2l^2kT \tag{31}$$

surface diffusion

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \Omega\delta D_\mathrm{gb}(B\sigma^2 C - \gamma_\mathrm{gb} - \sigma l)/4Ll^2kT \tag{32}$$

grain boundary diffusion

where Ω is the atomic volume, N_a is the number of atoms per unit area, D_s is the surface diffusion coefficient, δ is the grain boundary width, D_gb is the grain boundary diffusion coefficient, l is the crack tip radius, L is approximately half the grain size, and k and T

have their usual meaning and $B = \pi(1 - \nu^2)/2E_{\mathrm{ef}}$. The uniqueness of Steven's and Dutton's approach is that the thermodynamic stability of any crack can be calculated only if the local stress at the tip of the crack is known.

Substitution of the above equations into Equation 13 yields the creep rates by crack growth

$$\dot{\varepsilon}_{\mathrm{p}} = \frac{12(1 - \nu^2)V\sigma}{\pi R E_0}(1 + s/R)^2 \times \left(1 - \frac{3}{(7 - 5\nu)(1 + s/R)^5}\right)\Omega^2 N_{\mathrm{a}} D_{\mathrm{s}} \times (B\sigma^2 - \gamma_{\mathrm{gb}}/C)/2l^2kT \quad \text{surface diffusion} \tag{33}$$

$$\dot{\varepsilon}_{\mathrm{p}} = \frac{12(1 - \nu^2)V\sigma}{\pi R E_0}(1 + s/R)^2 \times \left(1 - \frac{3}{(7 - 5\nu)(1 + s/R)^5}\right) D_{\mathrm{gb}}\Omega\delta \times (B\sigma^2 C - \gamma_{\mathrm{gb}} - \sigma l)/4Ll^2kT \quad \text{grain boundary diffusion} \tag{34}$$

Alternatively, assuming that the crack opening is entirely due to uniform stress acting at infinity, without taking into account the effect of a local stress concentration due to the presence of pores, Equations 33 and 34 assume the form

$$\dot{\varepsilon}_{\mathrm{p}} = \frac{36(1 - \nu^2)VC\sigma}{2\pi E_0 R^3 l^2 kT}\Omega^2 N_{\mathrm{a}} D_{\mathrm{s}}(B\sigma^2 - \gamma_{\mathrm{gb}}/C) \quad \text{surface diffusion} \tag{35}$$

$$\dot{\varepsilon}_{\mathrm{p}} = \frac{36(1 - \nu^2)VC\sigma}{4\pi E_0 R^3 L l^2 kT}\Omega\delta D_{\mathrm{gb}}(B\sigma^2 C - \gamma_{\mathrm{gb}} - \sigma l) \quad \text{grain boundary diffusion} \tag{36}$$

Evidently, at high stress levels such that the quadratic in stress exceeds those linear in stress, for both diffusional mechanisms

$$\dot{\varepsilon} \propto \sigma^3 \tag{37}$$

which indicates that creep by crack growth is non-linear.

Further, it can easily be shown that when the crack length is greater than a critical value and the sum of the strain energy per atom at the crack tip and the chemical potential due to curvature at the crack tip is positive the crack will grow and sharpen by mass transport process. If, however, the surface energy term is larger than the strain energy term in Equation 33, the crack will spheroidize and shrink as vacancies diffuse away from the crack tip. As pointed out earlier, here we are concerned with the case when the strain energy term due to stress concentration is higher than the surface energy term and the crack propagates without the help of mass transport. This is considered to be the dominant mechanism of crack growth at low and intermediate temperatures.

5. Discussion

Creep by crack growth is considered to be of great importance because it may be the only mechanism of creep deformation operative at low and moderate temperatures. For example, in the case of aluminium oxide, Hasselman *et al.* [29] found the existance of creep by crack growth at temperatures at which diffusional creep mechanisms were negligible.

Furthermore, the elastic creep by crack growth may also be important in interpreting data in a single-loading condition. As soon as the load is applied on a brittle solid, crack extension will occur and this will be reflected in the change of its elastic response. This can be of particular significance with thermally anisotropic polycrystalline solids where spontaneous cracking may occur even without the application of external stress. The residual stress generated due to the difference in thermal expansion coefficients in different directions within a single grain, may greatly assist in promoting crack growth at stresses much lower than the fracture stress. This phenomenon will further be reinforced by a coarse-grained structure resulting in a grain size effect opposite to that predicted from Nabarro–Herring and Coble creep.

In the present analysis, the grain size dependence of elastic creep can be accounted for simply by equating the inherent flaw size, S, with the grain size, Γ, i.e. $s = \Gamma$. When this is done, a strong dependence of elastic creep on grain size is predicted (Fig. 2). Such strong effect of s/R or Γ/R on elastic creep by crack growth indicates again the importance of pores and stress concentration associated with pores in interpreting creep data in porous solids.

In addition to flaw (pore) size dependence of creep, a strong effect of pore volume fraction is also evident in Fig. 2. In the presence of pores, crack propagation would generally occur when the stress concentration at the radial (annular) cracks reaches a critical value. Such local stresses would be reached at more localities at higher applied stresses, and so the localized crack extension would become an important stress relieving mechanisms. This would allow increasing proportions of elastic creep strain to occur with increasing stress. Accordingly, the slope of the stress–strain rate curve would gradually increase with stress up to the fracture stress of the material in much the same way as the strain rate increases with crack length, s/R, predicted in Fig. 2. Such strong effect of porosity on creep is evident from the work of Fryer and Roberts [30] who observed two different stress exponents for two aluminas with different porosity. Coble and Kingery [31] also observed an increase of creep rate by a factor of 30 from 0 to 50% porosity. These results are in excellent agreement with the present analysis and in particular with Equations 35 and 36 which show a strong strain rate dependence of pore volume fraction and s/R. It is believed that, in this particular set of experiments, pores and therefore stress concentration associated with pores, must have played the dominant role in controlling the creep rate. In addition to supporting the crack opening displacement concept when interpreting the creep data in porous solids advanced in the present analysis, the reported results [31] also imply that the full potential of a material, in terms of its resistance to creep, can be realized only at theoretical density.

In analogy to the effect of stress concentration due to the presence of pores on strength [19], some small porosity is expected to have no significant effect on elastic creep in systems where the radial or annular cracks are much longer than the pore size itself. This behaviour is likely to be encountered in coarse-grained or highly cracked structures for which s/R is large.

It is clear from the present analysis that the creep rate by crack growth is directly proportional to the initial crack density. In systems where the nucleation of new cracks during stressing is less likely to occur, as may be the case with isotropic fine-grained materials free of internal stresses, the slope of the stress–strain rate curve is expected to be somewhat different from that in anisotropic solids. In thermally anisotropic polycrystalline solids, for example, the residual stresses may promote the initiation of new cracks at comparatively lower applied stresses and the slope of stress–strain rate curve would gradually, rather than abruptly, increase with stress up the fracture stress on the materials. In such cases, the number of cracks responsible for creep and the level of applied stress become dependent parameters. This has led many investigators to interpret the strain rate–stress curve in terms of two regions: one at low stresses where nucleation of new cracks and the rate of crack extension are minimal, and the other at high stresses where the crack nucleation and extension are much higher. The transition regime where the strain rate–stress curve changes its slope due to a drastic change of rate of crack nucleation and its extension is expected to be governed by the pore size, the pore volume fraction, the grain size and the pore and grain size distribution. In general, the transition regime is expected to move to lower stresses at larger grain sizes and at higher porosity volume fractions. The results of Warshow and Norton [15] for polycrystalline alumina, who reported stress exponent of unity at small grain, but at large grain sizes found a stress exponent of four may be interpreted in this manner.

6. Conclusions

Crack opening displacement concept is found to be a useful approach when interpreting the creep data in porous solids. It is shown that the presence of pores not only increases the number of crack precursors but also affects the crack opening displacement. As anticipated, the creep rate due to stress concentration outside a circular cavity is shown to be 2 to 3 times larger than the corresponding creep rate in the presence of straight isolated cracks unattached to pores. The effect of stress concentration is accommodated by the s/R which takes into account the effect of inherent (radial) flaw size (s) and the pore size (R).

Further, it is predicted that the creep rate by crack growth is also assisted by the presence of elastic and thermal anisotropy. In coarse-grained anisotropic ceramics, the number of crack precursors and the crack extension rate are expected to be greatly increased during stressing due to the additive nature of the applied and anisotropic stresses at certain localities. In fine-grained ceramics, where anisotropic stresses are minimal, no sharp transition from low to high stress exponent regimes are anticipated. Consequently, from the point of view of creep resistance, high density, fine grained, isotropic ceramics free of internal cracks are expected to exhibit highest resistance to creep deformation under high-stress low-temperature regimes.

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